ÜT II Formelsammlung

Idealer Abtaster

$$y(t) = x(t) \cdot \sum_k \delta(t - kT_A) \qquad \sum \delta(t-kT_B) \circ\!\!-\!\!\bullet w_A \cdot \sum \delta(w - kw_A)$$

$$\downarrow$$

$$Y(w) = \frac{w_A}{2\pi} \cdot \sum_\nu X(w - \nu w_A)$$

Nichtidealer Abtaster:

(Haltestufe)

$$d(t) = \sum_k g_0(t - kT_A) = g_0(t) * \sum_k \delta(t - kT_A)$$

$$\downarrow$$

$$D(w) = G_0(w) \cdot w_A \cdot \sum_\nu \delta(w - \nu w_A) = w_A \cdot \sum G_0(\nu w_A)\, \delta(w - \nu w_A)$$

$$\hookrightarrow \quad y(t) = x(t) \cdot d(t)$$

$$\downarrow$$

$$Y(w) = \frac{1}{2\pi} [U(w) * D(w)] = \frac{w_A}{2\pi} G_0(w) \cdot \sum_\nu X(w - \nu w_A)\, \delta(w - \nu w_A)$$

$$= \frac{w_A}{2\pi} \sum_\nu G_0(\nu w_A)\, U(w - \nu w_A)$$

Quantisierung + Codierung:

bei l Stufen: $M = ldL$ $\frac{bit}{Stufe}$

Mapper: M bits → 1 Symbol

Bitrate: $V_B = \frac{1}{T_B} = V_S \cdot ldL$ mit $V_S = \frac{1}{T} = 2f_N$ (Grenzfrequenz d. TP)

M-stufige Impulse

ldN $\frac{bit}{Symbol}$ → $V_S =$

Symbolfehler-Wkeit

$P_S(b_0)$ ~~$Q(\frac{b}{\sigma})$~~ V_B

4 QAM	$2Q(\frac{b}{\sigma}) - Q^2(\frac{b}{\sigma})$	$b = \frac{d}{\sqrt{2}}$	$2\frac{B}{1+\alpha}$
16 QAM	$3Q(\frac{b}{\sigma}) - \frac{9}{4}Q^2(\frac{b}{\sigma})$	$b = \frac{d}{3\sqrt{2}}$	$4\frac{B}{1+\alpha}$
2-PSK	$Q(\frac{b}{\sigma})$	$b = d$	$\frac{B}{1+\alpha}$
M-PSK/PAM	$2\frac{M-1}{M} Q(\frac{b}{\sigma})$	$b = \frac{d}{M-1}$	$\frac{B}{1+\alpha} ldM$
2-PAM	$P[a_1] Q[\frac{E-a}{\sigma}]$ $+ P[L] Q(\frac{b-E}{\sigma})$		

2-stufige PAM mit $a = k \cdot b$

$\downarrow$

$S = \frac{1}{2} b^2 (k^2 + 1)$

$P_S = Q\left(\frac{b(1-k)}{\sqrt{\sigma}}\right)$

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{1}{2}v^2}\, dv$$

Eig.: $Q(x) = 1 - Q(-x)$

$Q(\bar{x}) = 1 - Q(x)$

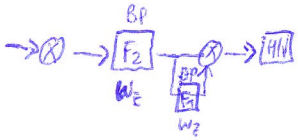
$Q(0) = \frac{1}{2}$

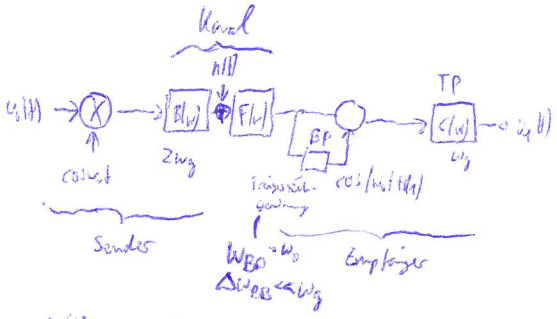
mittl. Symbolleistung:

$$M_{a_k} = \sum_\nu |a_\nu|^2 \cdot P[a_\nu]$$

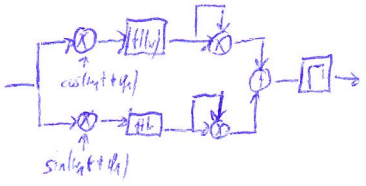
Zweiseitenband-AM

„Überlagerungsempfänger"





ohne Trägerfrequenzlage: (A 10)



$$U(t) = U_0(t)\cdot\cos\omega_0 t$$

$$\Rightarrow U(\omega) = \frac{1}{2}\left[U_0(\omega-\omega_0) + U_0(\omega+\omega_0)\right]$$

$U_0 \in \mathbb{R} \rightarrow U_0(-\omega) = U_0^*(\omega)$

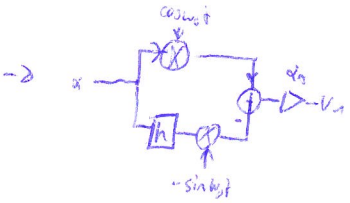
$\rightarrow |U_0(\omega)|$ gerade

$\arg\{U_0(\omega)\}$ ungerade

SSB-AM

$$U_1(\omega) = \frac{\alpha_0}{2}\left[X(\omega-\omega_0) + X(\omega+\omega_0)\right] + \frac{\alpha}{2}\left[X(\omega-\omega_0)\,\mathrm{sgn}(\omega-\omega_0) - X(\omega+\omega_0)\,\mathrm{sgn}(\omega+\omega_0)\right]$$

$$\Rightarrow u_1(t) = \alpha_A\cdot\left[x(t)\cos\omega_0 t \mp \hat{x}(t)\sin\omega_0 t\right] \quad \text{mit } \hat{x}(t) = x(t) * \underbrace{\frac{1}{\pi t}}_{h(t)}$$



$\sin\alpha\cdot\cos\alpha = \frac{1}{2}\sin 2\alpha$

$\cos^2\alpha = \frac{1}{2}(1+\cos 2\alpha)$

$\sin^2\alpha = \frac{1}{2}(1-\cos 2\alpha)$

$\cos\alpha\cdot\cos\beta = \frac{1}{2}(\cos(\alpha-\beta)+\cos(\alpha+\beta))$

$\sin\alpha\cdot\sin\beta = \frac{1}{2}(\cos(\alpha-\beta)-\cos(\alpha+\beta))$

$\sin\alpha\cdot\cos\beta = \frac{1}{2}(\sin(\alpha-\beta)+\sin(\alpha+\beta))$

$x(t) = x_g(t) + x_u(t)$

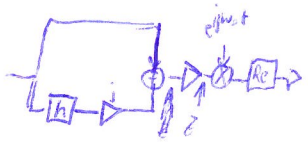
$x(-t) = x_g(t) - x_u(t)$

$x_g(t) = \frac{1}{2}(x(t)+x(-t))$

$x_u(t) = \frac{1}{2}(x(t)-x(-t))$

Hilbert-Transformation $\quad Z(\omega) = 0 \;\forall\; \omega < 0 \rightarrow Z(\omega) = \frac{1}{2}X(\omega)(1+\mathrm{sgn}(\omega))$

$= \frac{1}{2}X(\omega)(1+jH(\omega))$



$$z(t) = \frac{1}{2}x(t) + \frac{j}{2}\hat{x}(t) * h(t)$$

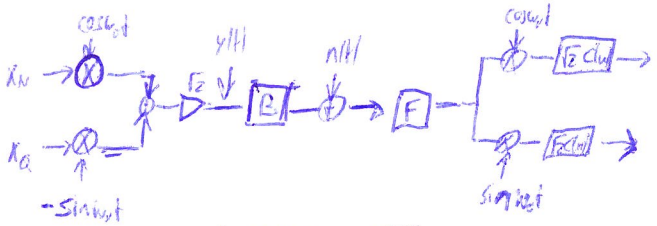
$H(\omega) = -j\,\mathrm{sgn}\,\omega = \frac{\mathrm{sgn}\,\omega}{j}$

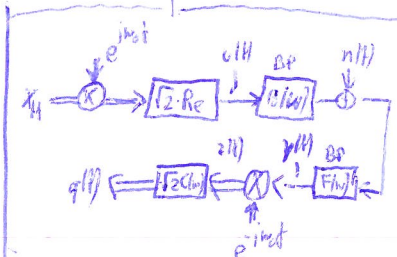
$h(t) = \frac{1}{\pi t}$

$x\cdot\sin\omega_0 t \xrightarrow{h} -x(t)\cos$

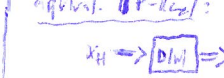
$\cos \xrightarrow{h} \sin$

QAM





äquival. TP-Kanal:



$D(\omega) = B(\omega+\omega_0)\cdot C(\omega)$

$$y(t) = \sqrt{2}\cdot x_N(t)\cos\omega_0 t - x_Q(t)\sin\omega_0 t$$

Komplexe Darstellung: $y(t) = \sqrt{2}\,\mathrm{Re}\{(x_N(t)+j\,x_Q(t))\,e^{j\omega_0 t}\}$ ($x_N(t)$)

IFFT: $\quad s_n = \sum_{\nu=0}^{N-1} z_\nu\, e^{j\frac{2\pi}{N}\nu n}$

Demodulation: $W = \frac{1}{\sqrt{N}}\left[\;\rightarrow$ S. 29

Filter

$f(t) \in \mathbb{R} \rightarrow F(-\omega) = F^*(\omega)$

$f(t)$ gerade $\Rightarrow F(\omega) \in \mathbb{R}$, gerade

$f(t)$ ungerade $\Rightarrow F(\omega) \in \mathbb{C}$, ungerade